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United Kingdom

Incorporated in the United Kingdom,

[ADP No. 07938194001]

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SECTION 30 (1977) APPLICATION FILED 05.10.2000

Patents ADP number *(if you know it)*

40507460007

If the applicant is a corporate body, give the country/state of its incorporation

United Kingdom

4. Title of the invention

"Increasing Data Transmission Bit Rates"

5. Name of your agent *(if you have one)*

"Address for service" in the United Kingdom to which all correspondence should be sent *(including the postcode)*

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Batchellor, Kirk & Co.

Date: 4th November 1999

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VERTICALBAND LIMITED

4. Title of the invention

"INCREASING DATA TRANSMISSION BIT RATES"

5. State how the applicant*(s)* derived the right from the inventor*(s)* to be granted a patent

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Patents ADP number *(if you know it)*: 07985294001

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Increasing Data Transmission Bit Rates

This invention relates to signal transmission systems, and in particular, to systems in which a continuously transmitted signal carries a series of discrete voltage levels. Thus it may be applied to communication systems or to any other system in which a multi-level input passes through a signal path having the characteristics of a finite response (FIR) filter. Such a system is referred to herein generally as an "ISI channel"

In conventional digital data transmission systems, signals are usually transmitted in a "binary" scheme, that is to say as two discrete voltage levels. However, it is possible to increase the data rate over a given channel, by transmitting higher order constellations, in which, instead of two distinct voltage levels, several voltage levels are used.

A communication system can be divided into three basic elements: transmitter, channel and receiver. The second element – the channel is the one that determines the upper limit of the communication bit rate. For binary transmission (e.g. transmission of +1V and –1V), when the bit rate is close to the channel bandwidth, internal noise is introduced. This noise, which is responsible for the upper limit on the bit rate, is called ISI (Inter Symbol Interference). Under the "ISI noise" the received bit voltage level is the sum of a scaled transmitted bit level plus noise, which is a linear combination of the surrounding bits of the received bit. Once the channel characteristics are known an inverse operation can be done, at the receiver, to minimise the ISI noise effect.

When the transmission rate is low enough (no ISI effect), the bit rate can be increased by transmission of higher order constellation. Then instead of a binary transmission the transmitter can send one of several different voltage levels each representing a symbol. In general, 2^n constellation order transmission, where every

symbol carries n bits, has n times higher bit rate than in the binary transmission. The bit rate can be thought as the area of a rectangle, where one axis is the symbol rate and the other axis is the log2 of the constellation size (voltage axis).

Normally a training sequence is initially sent to the receiver. This training sequence, in effect, provides the receiver with information about the channel parameters.

Unfortunately, however, the use of a training sequence implies a delay before proper transmission can begin, and if the nature of the communication channel changes frequently, as is the case (for example) in the case of mobile telephone systems, it is necessary to repeat the training sequence regularly, which slows down the total possible rate of data communication, so requiring a wider bandwidth or lower bit rate. Also, in a situation where a receiver begins to receive a transmission which has already started (e.g. digital TV), the training sequence will not necessarily be available.

Accordingly, the present invention seeks to provide a method of enabling high order constellation communication, without lowering the symbol rate, by evaluating the channel characteristics without the use of a specific training sequence.

High order constellation communication through ISI channels has the property that the ISI noise effect is much larger than at lower order constellation. The high order constellation acts in effect, as an amplifier on the basic binary ISI noise level. This high level noise prohibits the estimation of the channel characteristics under high order constellation in known systems.

Accordingly, the present invention provides a method of high order constellation signal transmission through an ISI channel, by identifying “reliable symbols”, that is to say symbols which are closer to their received constellation points (i.e. their scaled transmitted symbols free from ISI noise effect symbols) than to any other received constellation point. The constellation may be of any size and the channel may be up to the binary open eye channel limit

$$(\sum_{i \in ISI} |i| \le 1).$$

Since the ISI noise results from a combination of the ISI coefficient values and the surrounding symbols, the method of the invention preferably comprises identifying a symbol having surrounding symbols of a relatively lower level, up to some predetermined distance before and after the identified symbol, nominating the identified symbol as a "reliable symbol", and repeating the process a number of times along the received signal sequence, in order to gather a sufficient number of reliable symbols to enable a proper evaluation of channel characteristics to be achieved.

Preferably, therefore the method of the invention includes the step of estimating the constellation points location, so as to establish the channel gain, and then estimating the ISI coefficients so that a suitable inverse operation can be carried out in order to minimise the ISI noise.

One embodiment of the invention will now be described by way of example, with reference to the accompanying drawings in which:

Figure 1 is a functional block diagram of the method of the invention;

Figure 2 is a diagram illustrating a first sequence of transmitted symbols;

Figure 3 is a diagram illustrating a second sequence of transmitted symbols;

Figure 4 shows a 256 QAM signal constellation;

Figure 5 shows the algorithm for constellation points location estimation; and

Figure 6 shows the algorithm for channel parameters (ISI coefficient) estimation.

Referring to Figure 1, a signal constellation 4 is applied to a QAM modulator 2 so as to produce an output which is applied to an ISI channel 6. The resultant transmitted constellation after demodulation, as indicated at 8, consists of constellation points which have been shifted by the ISI channel, and the output from the demodulator 10 is applied to a reliable symbols extractor 12 which produces two outputs. One output, indicated at 14, consists of the extracted reliable symbols, whilst the other output 16 consists of the

related surrounding symbols for each reliable symbol, and these two signals are applied to a channel parameter estimator 18. This in turn produces two parallel outputs, one output 20 consisting of ISI coefficients, whilst the other output 22 consists of the actual constellation points.

These two outputs 20 and 22 are applied to the input of an inverse channel filter 24, another input of which receives the output of the demodulator 10 directly, on line 26. Consequently, the inverse channel filter is able to produce a corrected symbols constellation 28, by applying the derived inverse channel characteristics to the symbols constellation received on line 26.

The principle of operation of the reliable symbols extractor is illustrated in Figures 2 and 3, each of which illustrate a series of transmitted and received symbols.

Referring to Figure 2, this illustrates the transmission of a series of different symbols 30, 32 ..., 40, 42, having voltage levels +11, -5, +7, +7, +11, -3, and +13 respectively. Thus each different transmitted symbol, in the case illustrated, is represented by a different voltage level, the voltage levels being separated by two volts, the maximum being 13 volts.

A received central symbol is illustrated at 44, and it will be seen that this has an amplitude of just under 13 volts, but this amplitude is not likely to be "reliable", because the surrounding symbols 30, 32, 34 and 38, 40, 42 are of relatively high amplitude, and in fact, of course, the received symbol 44 really represents the transmitted symbol 36 but has a considerably greater apparent amplitude.

By contrast, Figure 3 illustrates the situation in the same ISI channel, where a series of low amplitude transmitted signals, 46, 48, 50, 52, 54 and 56 surround a relatively high amplitude signal (7V) 58. In these circumstances, the received symbol 60 is much closer in amplitude to the signal 58 than any other possible constellation point, because the inter symbol interference is at a lower level, as a result of the lower amplitudes of the surrounding symbols. Thus it is identified as a reliable symbol

Since the ISI channel can be characterised by a channel gain and a series of coefficients operating on the symbols surrounding the "reliable symbol", the channel characteristics can be derived if a sufficient number of reliable symbols can be identified.

Figure 4 shows (by way of example) a 256 QAM signal constellation.

Figure 5 shows the algorithm for constellation points location, while Figure 6 shows the algorithm for channel parameter estimation. The following analysis presents a model for the noise that is generated from the combination of ISI and high order constellation.

2

Model of the noise[1] under ISI and high order constellation

To present the "Reliable Symbols" theorem and the following different estimation methods, a model for the noise that is generated from the combination of ISI and high order constellation is derived.

The Standard Deviation (STD) of the ISI noise under high order constellation is:

$$E(n_{isi}^{\ 2}) = E(\sum_{\substack{i=n-b \\ i\neq n}}^{n+f} x_i \cdot ISI_{i-n})^2 = E(\sum_{\substack{i=n-b \\ i\neq n}}^{n+f} \sum_{\substack{j=n-b \\ j\neq n}}^{n+f} x_i \cdot x_j \cdot ISI_{i-n} \cdot ISI_{j-n})$$

where x_n is a transmitted symbol, i.i.d with uniform distribution, and ISI_n is an ISI coefficient.

Then ,

$$E(n_{isi}^{\ 2}) = \sum_{\substack{i=n-b \\ i\neq n}}^{n+f} E(x_i^2) \cdot ISI_{i-n}^{\ 2} = E(x^2) \cdot \sum_{\substack{i=-b \\ i\neq 0}}^{f} ISI_{i-n}^{\ 2} = \sigma_x^{\ 2} \cdot \|ISI\|^2 \tag{2.1}$$

Easy to show that $E(n_{isi}) = 0$.

Eq.(2.1) shows that the high order constellation is acting in effect as an amplifier on the basic STD of the ISI. Fig.(2.1) shows the STD of the ISI noise effect as a function of the size of the constellation. We'll use the pdf derivation of the sum of independent random variables [1] to find the pdf for a weighted sum of independent identical uniform distribution variables. For the weighted sum:

$$n_{isi}^{n} = \sum_i ISI_{i-n} \cdot x_n \quad : \quad i \in [n-b, n+f], i \neq n$$

the characteristic function of n_{isi}^{n}, is:

$$\begin{aligned} \Phi_{n_{isi}}(\omega) &= E(e^{j\omega n_{isi}^{n}}) = E(e^{\sum_i ISI_{i-n} \cdot x_i}) \underset{iid}{=} E(e^{ISI_{-b} \cdot x_{n-b}}) \cdots E(e^{ISI_f \cdot x_{n+f}}) \\ &= \Phi_{X1}(\omega) \cdots \Phi_{X2k}(\omega) \end{aligned} \tag{2.2}$$

[1] We refer as "noise" to the ISI effect on the signal. AWGN refers to the gaussian noise

where $X_i = ISI_{i-n} \cdot x_i$.

Based on the property that the characteristic function is the Fourier transform of the pdf function f, the pdf of n_{isi} is:

$$f(n_{isi}) = f_{X1} * f_{X2} * \cdots * f_{X2k} = f(ISI_{-k} \cdot x) * f(ISI_{-k+1} \cdot x) * \cdots * f(ISI_k \cdot x) \qquad (2.3)$$

For the case where all the ISI coefficients are identical, the pdf of n_{isi} is k-1 convolution of identical rectangular windows. When the ISI coefficients are not identical it's easy to show that the pdf of n_{isi} is identical, in the second derivation

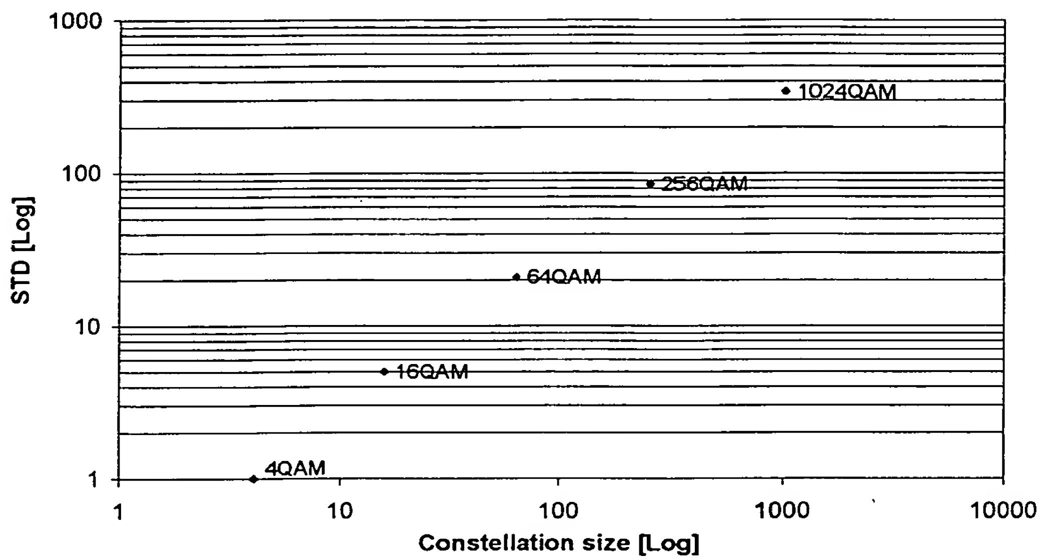

Figure 2.1: *STD of the ISI effect as a function of the constellation size.* $(\|ISI\|^2 = 1)$

sign and similar in shape, to the pdf with identical coefficients. $f(n_{isi})$ becomes closer to the pdf of the gaussian process as a function of k (in a finite size range). Fig.(2.2) shows the pdf of n_{isi} for ISI with $k=3$ where the coefficients are identical and as a comparison the gaussian distribution. Notice that while the central limit theorem [1] ensure that $f(n_{isi})$ has a gaussian pdf for infinite k, as low as k=3 where the pdf of n_{isi} is a function of $\pm x^2$ (minus for the central part) $f(n_{isi})$ is already very close to the gaussian pdf.

Defining the range of magnitudes in each axis of the constellation to be unity, a uniform discrete distribution of the constellation points are dense enough, at high

order constellation, to be considered as continues uniform distribution. This means that the noise produced by high order constellation, uniform distributed communication, through an ISI channel can be modelled as a gaussian noise with zero mean and variance as in Eq.(2.1).

This gives the ability to use the SNR to model the noise level of the combine ISI – high order constellation, using the same scale as for the gaussian case.

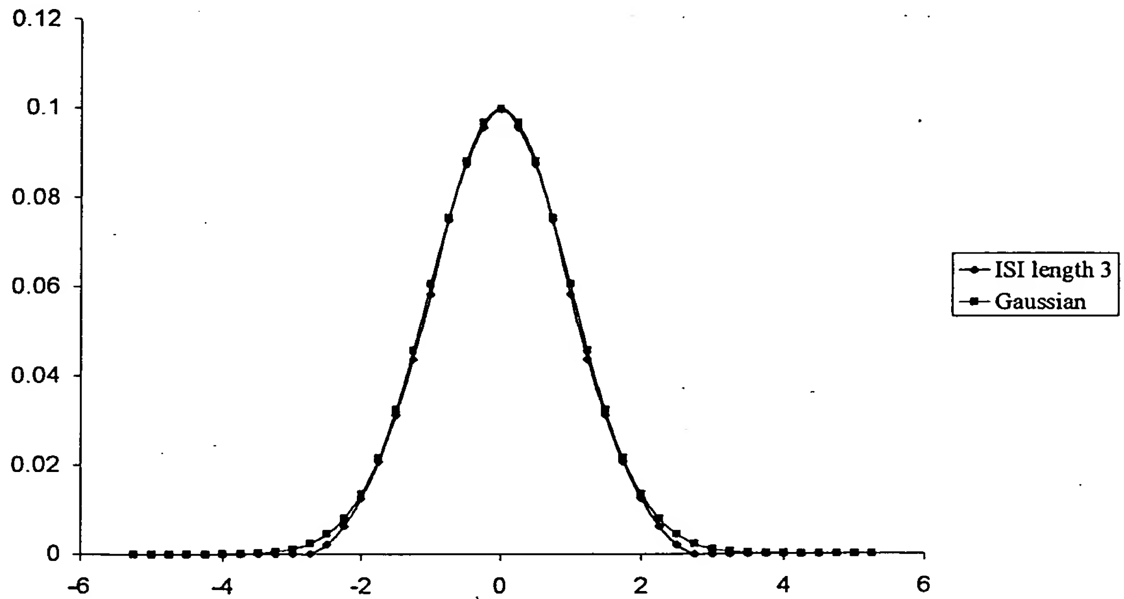

Figure 2.2: Comparison of the n_{isi} pdf, (ISI length –3) with the gaussian pdf

The SNR as a function of the ISI and the constellation size, is then:

$$SNR=\begin{Bmatrix} -7db & 16QAM \\ -13db & 64QAM \\ -19db & 256QAM \\ -25dB & 1024QAM \end{Bmatrix}+10\log\frac{1}{\|isi\|^2} \tag{2.4}$$

Eq.(2.4) emphasise the high level of ISI noise caused by the high order constellation "amplifier effect". In the next chapters we use the properties of the ISI- high order constellation noise that were developed in this chapter.

Reference

[1] A. Leon-Garcia, "Probability and random processes for electrical engineering," *Addison Wesley*, Second edition, September 1994

4

"Reliable Symbols" theory and Channel Parameters Estimations

Abstract

Blind detection under high order constellation and Intersymbol Interference (ISI) channel is a desired capability. Enabling high order constellation communication without lowering the symbol rate enables higher bit rate, for very important present and future communication scenarios. Examples for such scenarios are: one receiver for several transmitters, mobile phones and digital TV transmission. High order constellation is shown to have an amplifier effect on the level of the ISI channel affect. We present a new method, the "Reliable Symbols" method, which enables estimation of the channel parameters and the received constellation points under high order constellation and ISI channel. Based on these estimations, successful detection can be done using non-blind methods.

The "Reliable Symbols" method enables to identify received symbols that are closer to their constellation points, than to any other constellation point. An estimation of the received constellation points (the product of the transmitted constellation points and the channel gain) based on the "Reliable Symbols" method is presented. Using in addition to the reliable symbols, the surrounding symbols of every reliable symbol, an estimation of the channel parameters is presented. The "Reliable Symbols" method enables estimation for channels up to the binary open eye level ($\sum_{j\neq 0}|ISI_j| \leq |ISI_0|$), while there is no limit on the constellation size. Simulations confirm the "Reliable Symbols" method and the related estimations to be successful under channels up to the binary open eye level and for constellation up to at least 1024QAM.

4.1 Introduction

In this chapter a new method that enables successful parameters estimations under high order constellation communication through Intersymbol Interference (ISI) channels, is presented.

Channel bandwidth limitations motivate the use of high order constellation communication, which enables high bit rates. High order constellation communication through ISI channel, as it is shown, has a very high ISI noise effect, where the constellation size is acting in effect as an amplifier on the ISI noise level. Blind detection, which is based on channel parameters estimations, has been therefore a very difficult, if not impossible, task for high order constellation communication [1],[2]. Normally blind detection for high order constellation is done using blind equalization. Blind equalization is a mechanism that finds and implements the inverse filter for the channel impulse respond, which minimise some cost function. The cost functions that are involved depend on the statistics of the transmitted symbols. The simplest blind equalizer is the Reduce Constellation Algorithm (RCA) [1],[3]. The cost function minimised by the RCA is the Minimum Square Error (MSE) with respect to a reduce number of symbols, which normally are not a subset of the constellation points. The RCA is suitable for constellation size up to 32 points [1]. The most common equalization method is the Constant Modulus Algorithm (CMA) [4],[5],[6]. The cost function minimised by the CMA is the dispersion of the output of the equalizer around a circle. The CMA main advantage is that it does not requires carrier frequency recovery. The CMA is suitable for constellation size of up to 64 points [1]. The MultiModulus Algorithm (MMA) [1],[7], is based on the CMA but acts on each axis, in the QAM constellation, separately. The (MMA) is suitable for constellation size of up to 128 points [1]. The General MMA (GMMA) [1] is the latest algorithm, which in principle divides the full constellation to subsets and minimise the MMA cost function according to each subset. The GMMA was shown to be suitable for constellation size of 256 points [1]. All the mentioned algorithms do not address the problem of the constellation size amplifier effect. Consequently they are optimal just for 4QAM or PSK. The minimum of the different cost functions raise with the constellation size, which make them unsuitable for use in common ISI channels where just low order constellation communication operates successfully. In

addition the present algorithms require the knowledge of the transmitted points and the channel gain recovery.

We present a new method, which identifies symbols that are less affected by the ISI channel. The distances from their origin points, the received constellation[2] points, are less than half the distance of the nearest points in the received constellation. We call these symbols "Reliable Symbols", "Reliable" because the nearest constellation point to the received "Reliable Symbol" is its origin point. The different parameters estimations use then the reliable symbols for the estimation process. Two applications of parameters estimation under high order constellation communication through ISI channel, based on reliable symbols are presented: 1) Estimation of the constellation points, the product of the transmitted constellation points and the channel gain, and 2) ISI coefficients estimation. The estimations are shown to be suitable even for time varying channels. These parameters estimations enable to successfully detect the received signal by using equalization or/and by soft decision methods [8],[9],[10]. It is shown that the upper limit on channel ISI level is $\sum_{i\neq 0}|ISI_i| = 1$ (the binary modulation open eye channel condition. e.g. 4QAM), while there is no theoretical limit on the constellation size (estimation under up to 1024QAM is shown to be reliable by simulation).

The estimation is blind, in the sense that no training sequence is used. The required length of the data sequence is a function of the length and level of the ISI, and the constellation size.

Channel estimation using "Reliable Symbols" is suitable under variety of environments: Mobile phone channels where the ISI "window" length is short (about four coefficients) can be recovered effectively in "real time" for moderate constellation sizes. Data transmission over the telephone line, even under very high constellation size or/and long ISI window, where by using data storage capability the channel parameters can be recovered. Digital TV and other applications using VDSL, where the very high symbol rates enables to recover the channel parameters even for very high constellations or/and long ISI windows, effectively in "real time". High order constellation in software radio [11], where the transmitter and the channel parameters has to be recovered [12].

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[2] Scaled transmitted symbols because of the channel gain.

Channel and transmitter parameters estimations based on the "Reliable Symbols" method enable higher bit rate communication, by using higher order constellation, for a given limited bandwidth channel with symbol rate of up to the limit of the binary closed eye for 4QAM constellation.

The chapter is divided into seven sections: After the Section 4.2 presents the "Reliable Symbols" theory. Section 4.3 presents the constellation points estimation. Section 4.4 presents the ISI coefficients estimation. Section 4.5 presents adaptive estimation properties and Section 4.6 presents the simulation results and Section 8 contains the conclusions.

The following notations will be applied:

$x_n \equiv$ Transmitted symbols at time n.

$y_n \equiv$ The related received signal of x_n

$$|ISI| \equiv \sum_{\substack{i=-b \\ i\neq 0}}^{f} |ISI_i|$$

$ISI_0 \equiv 1$ (Normalised ISI).

$$\|ISI\|^2 \equiv \sum_{\substack{i=-b \\ i\neq 0}}^{f} ISI_i^2$$

$k=f+b$ (f-forward, b-backwards)

4.2 Reliable Symbols under high order constellation and ISI

In the estimation process normally the data points are considered to have the same importance when they are following the same statistics laws. In high order constellation communication under ISI conditions, a fraction of the data points are closer to their origin points, their received constellation points, than to any other received constellation points (Ch. 2). Let us define the received pulses that are closer to their origin points than to any other received constellation points, as "Reliable Symbols". The name, "Reliable Symbols", indicates that their metric from their closest received constellation points are the real metrics that are caused because of the channel noise. It is shown in the following sections, that knowing the right metric enables to recover or to estimate some key parameters in data communication as the ISI coefficients and the channel gain.

The problem is to find these precious symbols. Finding the whole set of reliable symbols is a very difficult if not impossible task. Instead we'll find data points from the reliable symbols set, where this subset is normally a small fraction of the reliable symbol set.

In high order constellation communication under ISI channel the metric of every received point depends on the ISI coefficients, the "future" and the "past" symbols (the surrounding symbols) and the adaptive gaussain noise:

$$d_n = \sum_{\substack{i=n-b \\ i \neq n}}^{n+f} x_i \cdot ISI_{i-n} + n_n \tag{4.1}$$

where, $\{x\}$ is the surrounding symbols sequence and n_n is an Adaptive White Gaussian Noise (AWGN) process.

For the moment let us assume high SNR so we can ignore the AWGN. Between the first two parameters x_i and ISI_{n-i} just the first, the surrounding symbols, can be considered as controllable. In general as the energy of the surrounding symbols are smaller the metric of a received data point from its origin point is smaller. Let us examine the case where the surrounding symbols have the same sign as the related ISI coefficients. In this case the metric is:

$$d_{\lim} = \sum_{\substack{i=n-b \\ i \neq n}}^{n+f} |x_i| \cdot |ISI_{i-n}| \tag{4.2}$$

This situation is the "worst case" scenario, where the surrounding symbols fulfils:

$$\{x_i : \sum_{\substack{i=n-b \\ i\neq n}}^{n+f} |x_i| \cdot |ISI_{i-n}| \le d_{\lim}\}$$

then, the metric of the related received data (the central symbol) from its origin point is ensured to be smaller than $d_{\lim}$.

Normally in blind detection the distribution of the ISI coefficients are not known. Therefore to ensure that symbols are under some reliable condition, the ISI is assume to be in the worst case, where its coefficients are distributed uniformly and equal the maximum one. Following a restriction on the sum of the absolute of the ISI noise:

$$\sum_{\substack{i=-b \\ i\neq 0}}^{f} |ISI_i| \le |ISI|_{\lim} \tag{4.3}$$

provides us with a criterion to find the reliable symbols.

Criterion: A symbol is from the Reliable Symbol subset if (not if and just if) its surrounding symbols fulfils:

$$|x_i| \le \frac{d_{\lim}}{|ISI|_{\lim}} \quad \forall i \in [n-b, n+f], i \notin n \tag{4.4}$$

where $d_{\lim}$ is half the distance between the two nearest received constellation points.

The number of detected reliable symbols for a given $d_{\lim}$ is obviously a monotonically increasing function of the probability,

$$P_R\{|x_i| \le \frac{d_{\lim}}{|ISI|_{\lim}}\} \tag{4.5}$$

which is a monotonically increasing function of $\frac{d_{\lim}}{|ISI|_{\lim}}$. For a given $d_{\lim}$, the number of reliable symbols is a monotonically decreasing function of the constraint on the ISI. For further presentation of the reliable symbols theory we'll use the common QAM (or the similar CAP) as a demonstration vehicle. Furthermore, we assume that the ISI effect is orthogonal, the noise in I and Q directions is a function just of symbols from the I and Q sequences respectively (in 4.2.3 the case where the ISI have complex coefficients is treated). Let us define two units as the distance between two nearest transmitted symbols in the I direction (the same is for the Q direction). Then, according to the definition of the reliable symbols, a reliable symbol has a metric of up to one unit, $d_{\lim}=1$, from its origin transmitted point. Then the maximum of the absolute of the ISI coefficients cannot exceed one. The proof is straight forwards:

From Eq.(4.4) maximum$|ISI|$ is abled when $\{x\}$, the surrounding symbols, have minimum magnitude, thus $\{|x|\}\equiv 1$. Then according to Eq.(4.4) when $d_{\lim}=1$,

$$|ISI| = \frac{d_{\lim}}{|x|_{\min}} \leq \frac{1}{1} = 1 \qquad (4.6)$$

Notice that one as an upper limit to the ISI noise effect is the definition of an open eye in the binary scheme modulation which exist as an example in 4QAM and 2ASK. This enables us to define an engineering oriented sentence:

Sentence: For channels where the binary modulation scheme has the property of an open eye, Reliable Symbols could be found for high order constellation communication.

The open eye limit is not a hard limit. Even in the case where the ISI channel exceeds the open eye limit reliable symbols could be found, but in addition not reliable symbols, mistakenly, could be considered as reliable. This error effect increases as the $|ISI|$ increases. Conceptually it can be considered similar to the Error - Rate function [18] where the rate exceeds the capacity point, the error cannot be any more zero, and it is growing as a function of the rate.

4.2.1 The probability to find a reliable symbol

To have an indicator to the data sequence length that is required for the different estimation process based on the reliable symbols, we derive the probability function to find a reliable symbol.

For the most general case where the only restriction is the open eye restriction, thus $|ISI| \leq 1$ the probability to find a reliable symbol is:

$$P_R = 2 \cdot \left(\frac{2}{\sqrt{constellation}} \right)^k \quad : |ISI| \leq 1 \qquad (4.7)$$

Proof: to ensure reliable symbol all the surrounding symbols must be from the lowest level of the constellation points, thus two points among the I (or Q) constellation points. This probability must be powered by the length of the ISI window to insure that every surrounding symbol that participates in the ISI effect is from the lowest points. The multiplication of two is because the points are gathered independently from both I and Q axes.

For lower upper limits to the absolute of the sum of the ISI, $|ISI|$, the probabilities to find reliable symbols become:

a. $$P_R = 2 \cdot \left(\frac{2}{\sqrt{const}}\right)^k \cdot (k+1) \quad : |ISI| \leq 0.5 \tag{4.8}$$

It is easy to show that for the 0.5 limit, in addition to the first layer of points in the constellation, the surrounding symbols can be with one symbol from the second layer of the constellation points.

b. $$P_R = 2 \cdot \left(\frac{4}{\sqrt{const}}\right)^k \quad : |ISI| \leq \frac{1}{3} \tag{4.9}$$

For 0.33 limit, the surrounding symbols can be from both the first and second layer of the constellation points.

c. $$P_R = 2 \cdot \left(\frac{4}{\sqrt{const}}\right)^{k-1} \left(\frac{2k+4}{\sqrt{const}}\right) \quad : |ISI| \leq 0.25 \tag{4.10}$$

For 0.25 limit, one of the surrounding symbols can be in addition to the first and second layers, in the third layer of the constellation points.

The basic idea for all three proofs is to find all the possible arrangements of the surrounding symbols for every given upper limit.

4.2.2 Reliable Symbols under unknown transmitted symbols

In blind detection where no training sequence is available, the receiver in the stage of the parameters estimation process, does not know what are the surrounding symbols. There are two ways to deal with this: 1) to assume a certain surrounding symbols sequence and 2) to select the symbols of the surrounding sequence according to their received energy. The first way can be implemented when the parameters estimations is done in parallel to some trellis decoding. The second way, the more broad solution, can be implemented for the parameters estimation just after the demodulation to an I-Q plan. For the parameters estimations that are addressed, we'll use the second approach. The surrounding symbols selection by their received energy, raises some difficulties that will be treated in section 4.5.

4.2.3 Complex ISI coefficients

Reliable symbols can be found even when the ISI channel operates in a not orthogonal way on the I and Q axes. In the more general form where the ISI coefficients are complex, the I axis surrounding symbols have an influence on the Q axis symbols (and the same for Q on I). The criterion on the surrounding symbols is to be under some energy limit, the real energy of the symbol, that is influenced from both the I and Q axis. As an example, for the QAM constellation, where $|ISI| \le 1$, the surrounding symbols have to be from the four lowest energy symbols, the lowest 4QAM in the QAM constellation. The probability to find a reliable symbol in this case is:

$$P_R = \left(\frac{4}{constellation} \right)^k \quad : \left| ISI_{complex} \right| \le 1 \tag{4.11}$$

which has $2 \cdot \left(\frac{\sqrt{constellation}}{2} \right)^k$ times smaller probability than the same limit with real ISI coefficients.

Reliable symbols can be found even under complex ISI noise, obviously it requires longer received data sequence length.

4.2.4 Reliable symbols with control errors

When the data sequence length is a critical parameter, where just relative short data sequence is available, under some assumption, a "noisy" reliable symbols detection can be done. "noisy" in the sense that in addition to the "real" reliable symbols the system wrongly defines not reliable symbols as reliable. The number of wrong reliable symbols can be controllable and made relatively small.

Based on the QAM demonstration vehicle, upper bound to the probability of a symbol to be detected wrongly as a reliable symbol is derived for the open eye limit case. If,

1. $\max(|ISI_i|) \le 0.25,\ i \in n,m \quad n,m \in k$

2. $\max(|ISI_i|) \le 0.1667\ \forall i \notin n,m$

and the criterion for selecting the surrounding symbols is according to the 0.5 limit instead of the 1 limit, then the upper bond on the probability for a wrongly detected symbol is:

$$P_R(error) = P(|ISI * x| \geq 1, x \in x') = P(x \in x') \cdot P(|ISI * x| \geq 1 \mid x \in x')$$

$$= \frac{k}{k+1} \cdot P(|ISI * x| \geq 1 \mid x \in x') \leq 2 \cdot P(|x_n| = 3) \cdot P\{sign(x_m \cdot ISI_m) = sign(x_n \cdot ISI_n)\}$$

$$\cdot P\{sign(\sum_{i \notin n,m} ISI \cdot x) = sign(x_m \cdot ISI_m)\}$$

$$+ P(|x_{q \notin n,m}| = 3) \cdot P\{sign(x_m \cdot ISI_m) = sign(x_q \cdot ISI_q)\}^2$$

$$\cdot P\{sign(\sum_{i \notin n,m,q} ISI \cdot x) = sign(x_q \cdot ISI_q)\}$$

$$= \frac{k}{k+1} \cdot (\frac{2}{k} \cdot \frac{1}{4} + (1 - \frac{2}{k}) \cdot \frac{1}{8})$$

$$P_R = \frac{k}{k+1} \cdot (\frac{1}{8} + \frac{1}{4k})$$

where x' is the ensemble of surrounding sequences under the 0.5 limit criterion, where one of the symbols is in the second layer of the constellation (the second layer under the same scale has magnitude of three).

The probability to find a reliable symbol is k+1 time higher than in the regular limit 1 reliable symbols detection. In other words, the required sequence length becomes substantial shorter when allowing controllable small error events.

4.2.5 Estimation under received symbols energies

Reliable symbols are found through a criterion on the surrounding symbols. The surrounding symbols are not known in this case, so the selection of the reliable symbols relays on the received surrounding signals energies. The surrounding received signals are also influenced by the ISI and their own surrounding symbols. The question that immediately raises is how do we know that a received signal with low level energy has a low-level transmitted energy as well? Generally we can't be sure, but when the level of the ISI is controllable, as in our case, following some guidelines as will be explained, a low level energy received signal has a very high probability to represent a low level energy transmitted symbol. A general proof, if possible, is very hard to derive. A motivation for the proof is described. We proved it empirically by computer simulations.

Proof motivation: A received signal has a low energy because: 1) Its origin point has low level energy, and its surrounding symbols have low level energies. 2) Its origin point has low level energy and its surrounding symbols have higher level energies,

then the received signal in half of the cases will approach zero or even change sign if the combination of ISI and surrounding symbols are high. 3) The origin point has high energy level and a special combination of relative high level energies surrounding symbols occurs. The last scenario is the problematic one, which may include surrounding symbols that are not under the required criterion. The probability of scenario 3 depends on the size of the ISI and dramatically on the criterion limit. The lower the limit the lower the number of combination for scenario 3, which means a smaller probability of scenario 3 to happened. In the cases when the ISI is known to be very short, as 2-4 coefficients at most, and the ISI or the constellation size are relative large, it is required to take more than just 2-4 reliable symbols to deal with scenario 3 effect (4-6 respectively, are recommended).

The central symbol influences on the surrounding symbols as well. In effect if:

$$sign(x_i \cdot ISI_i) = -sign(x_0)\,, \tag{4.12}$$

Then the effect of the central symbol on the surrounding symbol is:

$$effect(on\ y_i) = -sign(x_i) \cdot x_o \cdot ISI_{-i} \tag{4.13}$$

and obviously vice versa (concerning the signs),

where x_i is from the surrounding symbol and y_i is the related received signal. For the case where the ISI behaves as a symmetric FIR filter, thus $ISI_i = ISI_{-i}$ then when Eq.(4.12) is fulfilled, y_i becomes smaller in magnitude, or even change the sign (but for limit ISI effect it will still have a small magnitude). Therefore the probability of surrounding signals that fulfils Eq.(4.12) to pass the surrounding symbols criterion, is much higher than for those that do not fulfil it. Then it is easy to show that reliable symbols under symmetric ISI effect have lower magnitude on the average, than their origin points. (Actually the same situation happens for just symmetric in sign ISI). To deal with this effect the final constellation estimation is done with the subset s of reliable symbols. The subset s contains reliable symbols with lower energy, so that the probability to have symbols from both sides of the origin points is higher. The size of subset s depends on the ISI effect, as it increases the subset contains reliable symbols with smaller energy and vice versa.

4.3 Constellation points estimation (includes the channel gain)

In blind detection a major estimation parameter is the gain of the channel. The channel gain provides to recover the location of the transmitted points, the constellation points, on the I-Q plan. The detection process relies deeply on the constellation points estimation. Hard decision detection, or the variety of soft decision detection methods, use the distance between the transmitted point to the received point as a central parameter for detection. Communication through an ISI channel will result with symbols exceeding the extreme points of the transmitted constellation. In fact the received data points could reach as far as $1+\sum_i |ISI_i|$ times the maximum transmitted points (ignoring the gaussain noise, which with it the maximum received data point is even larger in magnitude).

For the estimation of the transmitted constellation points we assume no prior knowledge on the ISI coefficients. The estimation of the transmitted constellation points is done in two stages.

4.3.1 Initial estimation of the constellation points

Reliable symbols, by definition, have the property that the maximum distance from their origin transmitted points are, at most, half the distance of the nearest transmitted points in the constellation. Lets again define this distance to be one unit. After detecting "enough" received data points, we'll choose the maximum in magnitude in the I and Q axes from the reliable symbols set to be the initial maximum constellation point in each axis. Enough data points, according to the probability to find reliable symbols from the highest magnitude. Let $\hat{P}^1{}_{\max}$ be the initial estimation of the maximum constellation point, then:

$$\hat{P}^1 i = sign(i) \cdot \frac{\hat{P}^1{}_{\max}}{\sqrt{consttelation}-1} \cdot (2|i|-1) \tag{4.14}$$

is the estimation of the remaining constellation points in each axis, where i is the index from zero to each direction.

(We assume a rectangular QAM constellation. For different shapes the numerator in Eq.(4.14) is the number of constellation point for every axis minus one).

Notice that while the error in the maximum estimated points can be up to one unit, for points that are closer to zero the upper limit on the error goes down to,

$$|error|_{\min} \leq \frac{1}{\sqrt{constelltion} - 1} \quad : \hat{P}^1{}_{1,-1} \tag{4.15}$$

for the points with the smallest magnitude.

4.3.2 Final estimation of the constellation points

It is easy to show that,

$$\widetilde{P}^1{}_i = (2|i| - 1) \cdot \widetilde{P}^1{}_1 \tag{4.16}$$

when,

$$\widetilde{P}^1{}_i = \hat{P}^1{}_i - P_i \tag{4.17}$$

where P_i is the ith constellation point. Let us express the reliable symbol y^i_n by the origin point plus the related ISI noise coefficient d^i_n. Using Eq.(4.14,4.16,4.17) we obtain:

$$y^i_n = P_i + d^i_n = \hat{P}^1_i - \widetilde{P}^1_i + d^i_n = \hat{P}^1_i - (2|i| - 1) \cdot \widetilde{P}^1_1 + d^i_n \tag{4.18}$$

where n is the index of the reliable symbol and i the index of the related constellation point. In chapter 2 it is shown that the ISI noise, d^i_n can be modelled as a zero mean almost gaussian distribution parameter that does not depends on i. Using the weighted average estimator over $\hat{P}^1_i - y^i_n$, where $\{y\}$ is a subset s of the reliable symbols (Sec. 4.2.5), we get:

$$\hat{\widetilde{P}}_1 = \frac{1}{2|i| - 1} \cdot \left(\frac{1}{|s|} \sum_s \hat{P}^1_i - y^i_n\right) \tag{4.19}$$

where the expectation of $\hat{\widetilde{P}}_1$, is:

$$\begin{aligned} E[\hat{\widetilde{P}}^1_1] &= E[\frac{1}{2|i| - 1} (\frac{1}{|s|} \sum_s \hat{P}^1_i - y^i_n)] = E[\frac{1}{2|i| - 1} \{\frac{1}{|s|} \sum_s \hat{P}^1_i - (P_i + d^i_n)\}] = \\ &E[(\frac{1}{|s|} \sum_s \hat{P}^1_1 - P_1) + E(d_n) = \hat{P}^1_1 - P_1 = \widetilde{P}^1_1 \end{aligned} \tag{4.20}$$

then,

$$\hat{P}_i = \hat{P}^1_i + (2|i| - 1) \cdot \hat{\widetilde{P}}_1 \tag{4.21}$$

where $\{\hat{P}\}$ are the final estimation of the constellation points. The estimation in Eq.(4.19) is done on the first estimation error. Eq.(4.20) indicates that the estimation in Eq.(4.19) is not biased. Because $\{d\}$ is close to the gaussian process, and it is known that the averaging-estimator is optimal under gaussian process [15], the estimation in Eq.(4.21) can be considered as almost optimal.

4.4 ISI coefficients estimation

ISI coefficients estimation is a crucial stage in data detection under ISI channel. The ISI coefficients are essential for the recovering data process, using either equalization or soft decision detection as MAP [8],[9], or both. High order constellation communication under ISI channel, has a much larger noise level because of the amplifier effect caused by the constellation size (Chapter 2). By using reliable symbols, in effect, we bypass the high constellation noise effect. In other words, using reliable symbols enables to make the estimation under the same conditions as for the low order constellation case.

The estimation method is divided into two methods groups, when the ISI effect is known to be under $|ISI| \leq 0.5$ (or under a lower limit) and for the more general case where $|ISI| \leq 1$.

4.4.1 ISI coefficients estimation for $|ISI|$ upper limit under or equal to 0.5

The metric of a received signal from its origin, depends on the ISI coefficients, the surrounding symbols and the AWGN Eq.(2.1)). When the SNR is high and the surrounding symbols and the desired related metrics are known, then the ISI coefficients are simply a solution of k (the length of the ISI window) linear equations. The exact solution becomes to be the estimation when the surrounding symbols are noisy, then the optimal linear mean square solution is required. In our case, as was discussed before, the surrounding transmitted symbols are not known. Instead we use the received signal energies. For:

$$y_n = x_n + n_{isi} \tag{4.22}$$

where x_n is from a surrounding symbols set, y_n the related received signal and n_{isi} the related ISI noise effect. Under the assumption that a low level received signal

represent a low level surrounding symbol (Sec. 4.2.5), we'll use the general LS (weighted linear least square) solution for estimating the ISI coefficients:

$$\underline{ISI} = (\underline{\underline{X}}^T \underline{\underline{W}}\,\underline{\underline{X}})^{-1} \underline{\underline{X}}^T \underline{\underline{W}}\,\underline{\Delta} \tag{4.23}$$

where, X – n x k matrix where $n \geq k$, is the ensemble of the surrounding signals for each reliable symbol.

W – An n x n diagonal weight matrix.

Δ - n x 1 vector, is the metric of each reliable symbol from its origin.

Because n_{isi}, the noise on the surrounding symbols, has a close to gaussian distribution (as was shown in section 2) and the LS is the optimal estimation for the gaussian noise [15], Eq.(4.23) can be considered as close to optimal estimation of the ISI coefficients.

Under a certain constraint to the upper limit on $|ISI|$ (up to 0.5) at least k reliable symbols are chosen under the related criterion (4.2.1.a – 4.2.1.c), where k is the assumed upper limit on the ISI length. The metrics of the reliable symbols from their origin points creates the Δ vector. The related received surrounding signals values (magnitude and sign) creates the X matrix. Let us assume for the moment that the W matrix is a diagonal 1 matrix. The data sequence length is a function of the number of required reliable symbols.

Notice that for the case where the constellation is small enough where all the symbols are reliable, the estimation of Eq.(4.23) merges to the regular optimal LS solution. The rows of the X matrix are then, the surrounding received signals of sequential symbols, as all symbols are reliable. The Δ vector is then, the related metrics of the sequential symbols from their origins.

The constellation points estimation process uses the surrounding received signals, by choosing those that are under some magnitude limit. In the ISI coefficients estimation, beside of choosing the surrounding received signals to be under some magnitude limit, the estimation process uses the exact value of them (The X matrix). The quality and the limit of the estimation depend on the surrounding received signals noise level. From simulation it appear that the limit is $|ISI| = 0.5$. Above this limit the statistical LS estimation does not produces valid estimation. Under or equal to 0.5 limit, the quality of estimation, like the gaussian case, is a monotonically increasing function of the number of rows in the X matrix, the number of reliable symbols. Simulation shows

that from k+2 rows the estimation has a very small error for moderate SNR. The central symbol, the reliable symbol, has a major effect on the surrounding received signals (Sec. 4.2.5). By applying an upper limit to the energy of the reliable symbol, we reduce the noise on the surrounding symbols. The limit on the energy of the reliable symbols is a function of the assumed upper limit $|ISI|$. The probability to find a reliable symbol under the additional constraint then becomes:

a. For $|ISI| \le 0.5$ it was found that the reliable symbol magnitude can be up to the second layer of the constellation. The probability then becomes:

$$P_R = 2 \cdot \frac{4}{\sqrt{const}} \cdot \left(\frac{2}{\sqrt{const}}\right)^k \cdot (k+1) \quad : |ISI| \le 0.5 \tag{4.24}$$

b. For $|isi| \le 0.33$ and 0.25 it was found that the reliable symbol magnitude can be up to the third layer of the constellation. The probabilities then become:

$$P_R = 2 \cdot \frac{6}{\sqrt{const}} \cdot \left(\frac{4}{\sqrt{const}}\right)^k \quad : |ISI| \le \frac{1}{3} \tag{4.25}$$

$$P_R = 2 \cdot \frac{6}{\sqrt{const}} \cdot \left(\frac{4}{\sqrt{const}}\right)^{k-1} \left(\frac{2k+4}{\sqrt{const}}\right) \quad : |ISI| \le 0.25 \tag{4.26}$$

The best case is when the reliable symbols are limited to be just from the smallest constellation layer. Obviously under such constraint the required length of the data sequence has to be longer, two times longer for the 0.5 limit and three times for 0.33 and 0.25 limits.

In the case where the estimation of the ISI coefficients is based on the estimation of the constellation points, limiting the reliable symbol energy introduce another advantage. In Section 4.3 it was shown that error of the received constellation points estimation is smaller as the constellation points are closer to the zero. This means that reliable symbols under constraint of energy have a more accurate Δ vector in addition to a more accurate X matrix.

The W matrix operates as a diagonal weighted matrix [15]. Eq.(4.23) uses the W matrix to attach different weights for the different reliable symbols rows. As the reliable symbol is higher, up to its energy limit, the related weight value is lower. In such way the low energy reliable symbols that can produce better estimation, have larger weight in the estimation process than the higher energy levels symbols. Simulations have shown that values in the range of 0.7-1 are useful.

When the ISI channel is a symmetric FIR filter it is easy to show that the LS equation requires just half of the required rows, reliable symbols, although the length of the surrounding symbols has to remain the same. Fig.(4.1-4.3) shows the required data length

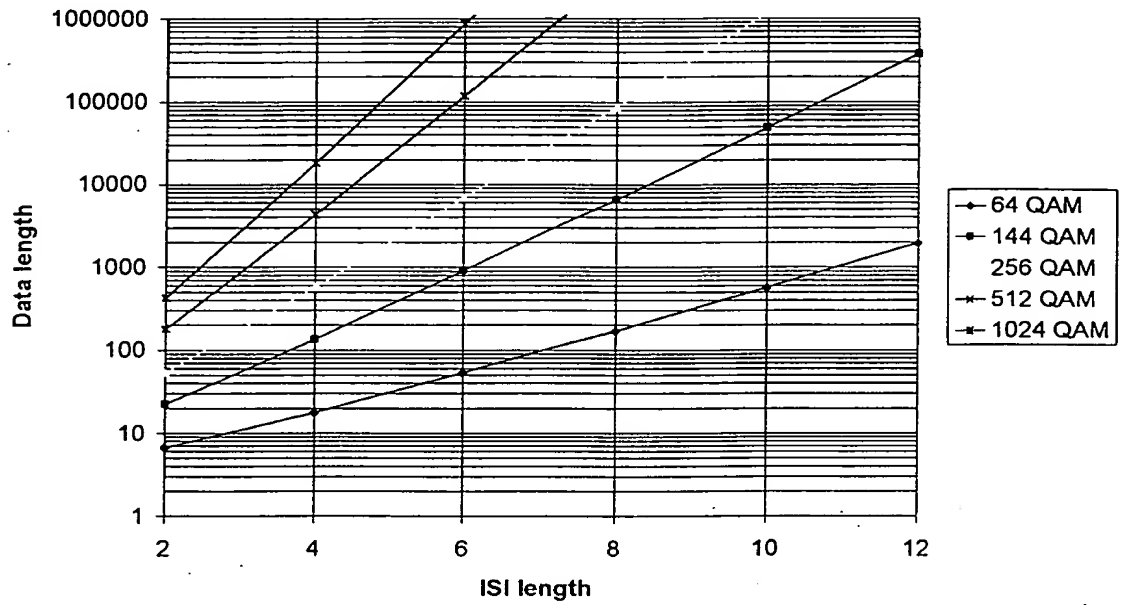

Figure 4.1: *Required data length vs. ISI length for different constellations.*

$|ISI| \leq 0.25$

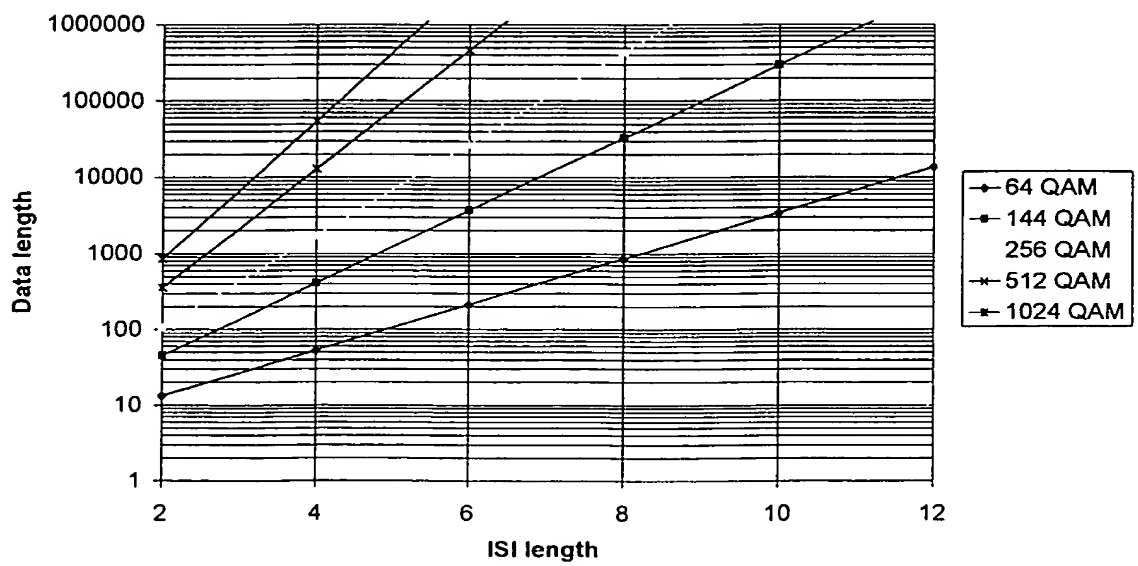

Figure 4.2: *Required data length vs. ISI length for different constellations.*

$|ISI| \leq 0.33$

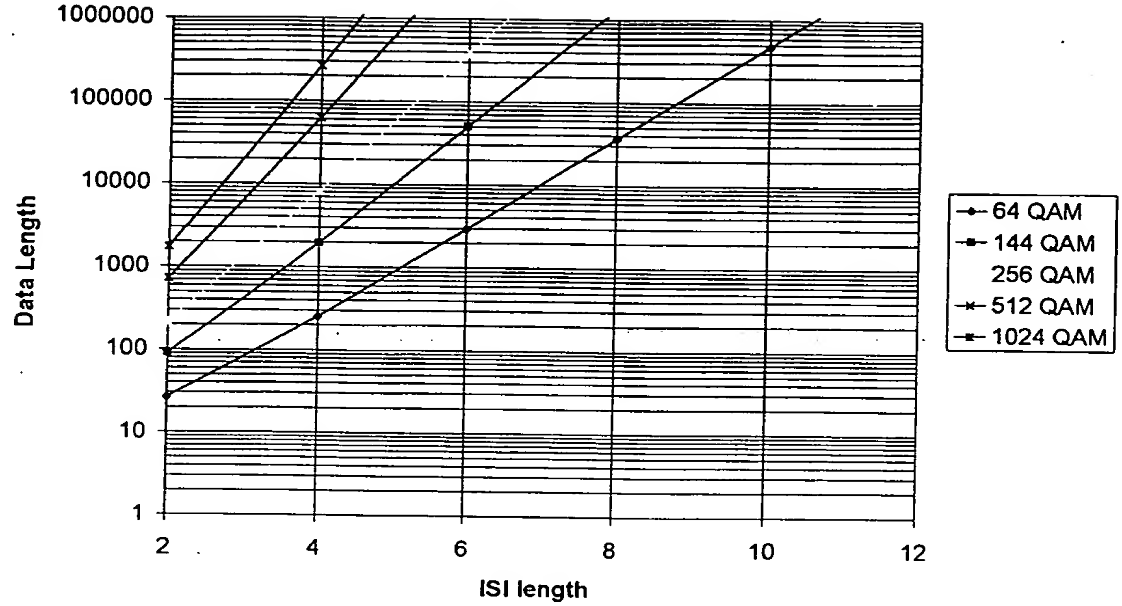

Figure 4.3: Required data length vs. ISI length for different constellations. $|ISI| \leq 0.5$

as a function of the ISI length and the constellation size, for several $|ISI|$ constraints, for the case the ISI channel is a symmetric FIR filter.

4.4.2 ISI coefficients estimation for $|ISI| \leq 1$

It is shown in Sec. 4.4.1 that when the ISI effect is high and the noise in the X matrix in Eq.(27) is high, successful estimation by using the statistical random processing of the LS method cannot be carried out. For the case where the $|ISI|$ is known to be smaller than unity, the length of the ISI is up to four coefficients and the ISI channel is a symmetric FIR filter, is practically important since it covers most of the mobile communication channels. The ISI coefficients estimation, using reliable symbols, for these kind of channels is now presented.

When the surrounding symbols are from the first layer of the constellation points and assuming high SNR, the reliable symbols can be placed between five discrete possible magnitudes (but not just there), lower than their origin points, with different probabilities, for each origin point. Because of the relative high level ISI the reliable symbols is limited to be just from the first layer of the constellation points. The five discrete magnitudes that reliable symbols can be placed in, the related surrounding

symbols set-up and the related probabilities are (for probability comparison we'll use the condition probability, where the surrounding symbols are assumed to be in the first layer. The probability on the sign is 0.5):

a. The scenario with no ISI noise:

$$x_{n-1} = -x_{n+1} \;:i \in [1,2] \;\Rightarrow\; y_n = x_n \tag{4.27}$$

The probability is 0.25 .

b. The scenario where two anti-symmetric symbols cancel their ISI effect:

1.

$$\begin{aligned} &x_{n-2} = -x_{n+2} \;\cap\; x_{n-1} = x_{n+1} \;\cap sign(x_{n-1}\, ISI_{-1}) = -sign(x_n) \\ &\Rightarrow y_n = x_n + 2\, ISI_1\, x_{n-1} \quad (|y_n| < |x_n|) \end{aligned} \tag{4.28}$$

2.

$$\begin{aligned} &x_{n-1} = -x_{n+1} \;\cap\; x_{n-2} = x_{n+2} \;\cap sign(x_{n-2}\, ISI_{-2}) = -sign(x_n) \\ &\Rightarrow y_n = x_n + 2\, ISI_2\, x_{n-2} \quad (|y_n| < |x_n|) \end{aligned} \tag{4.29}$$

The probabilities for both c.1 and c.2 is 0.25 .

c. The scenario where all symbols contribute towards zero:

$$\begin{aligned} &x_{n-i} = x_{n+i} \;\cap\; sign(x_{n-i}\, isi_i) = -sign(x_n) \;\; : i \in [1,2] \\ &\Rightarrow\; y_n = x_n + 2\, ISI_1\, x_{n-1} + 2\, ISI_2\, x_{n-2} \quad (|y_n| \le |x_n|) \end{aligned} \tag{4.30}$$

The probability is 0.125 .

d. The scenario where the related symbols to the higher ISI coefficients contributes towards zero and the other symbols contributes in the opposite way is:

$$\begin{aligned} &x_{n-i} = x_{n+i} \;\cap\; sign(x_{n-i}\, ISI_i) = -sign(x_n) \;\cap\; sign(x_{n-j}\, ISI_j) = sign(x_n) \\ &: i \in index(\max(|ISI_i|) \;\&\; j \neq i \\ &\Rightarrow\; y_n = x_n + 2\, ISI_i\, x_{n-i} - 2\, ISI_j\, x_{n-j} \quad (|y_n| \le |x_n|) \end{aligned} \tag{4.31}$$

The probability is 0.125 .

Notice that:

$$2 \cdot x_{\min} - (y_n^{28} + y_n^{29}) = x_{\min} \cdot |ISI| \tag{4.32}$$

Where y_n^{28} and y_n^{29} are the reliable symbols that fulfil Eq. (28) and (29) respectively.

The estimation algorithm:

1. Receiving "enough" data points.

   "enough" received data points according to the probability to find a suitable reliable symbol:

   $$P_R = 2\cdot\left(\frac{2}{\sqrt{constellation}}\right)^7$$

(where, the reliable symbol and its six surrounding symbols are from the first layer of the constellation points.)

2. Identifying a set of relibale symbols {y} (e.g. ten reliable symbols).
3. Calculating the ISI total effect by

   $$|ISI| = \frac{|data|_{max}}{|P_{max}|} - 1$$

   where $|data|_{max}$ is the value of the maximum in magnitude received data point, and $|P_{max}|$ is the value of the maximum in magnitude received constellation point.
4. Updating the received constellation points estimation:

   (This stage is for the case where the constellation points are from estimation.) Defining the closest reliable symbol to the smallest estimated received constellation point (in magnitude) as the new estimation of the estimated points that equal $|P|_{min}$. Then calculating the other constellation points in each axis to be:

   $$P_i = sign(i)\cdot|P|_{min}\cdot(2|i|-1): \; i = -k,..,-1,1,..k$$

   where, $2k$ is the number of constellation points in each axis.
5. Updating the ISI total effect:

   Defining the closest data point to $|x|_{min}\cdot(1-|ISI|)$ as the reliable symbols that fulfils condition 4.4.2.c. The distance between this point to the first constellation point is the updated $|ISI|$.
6. Finding all pairs of data that fulfil Eq.(4.32) (up to some small margin). Because there is a small probability for symbols that are not reliable symbols to fulfil Eq.(4.32), the set of pairs that appear most frequently, Y, is picked. Then defining y_S as the smallest, in magnitude, value from the set Y, and y_L as the largest, in magnitude, value from the set Y.
7. Estimating the magnitude of the ISI coefficients (without indexes):

The magnitude of the four coefficients of the ISI are calculated (because the assumption is a symmetric ISI channel, actually there are just two coefficient that are being estimated):

Using Eq.(28) and (29), the absolute of the two coefficients are calculated

$$|ISI|_{-m,m}=\frac{|P|_{\min}-y_S}{2\cdot|P|_{\min}}\ ,\ |ISI_{-n,n}|=\frac{|P|_{\min}-y_L}{2\cdot|P|_{\min}} \tag{4.33}$$

8. Finding the signs of the ISI coefficients:

   The surrounding symbols of the maximum, in magnitude, received signal, $|data|_{\max}$, are from the extreme points of the constellation. Because $|ISI|\leq 1$, the signs of the surrounding symbols do not switch in any case. The signs of the ISI coefficients are then the same as the signs of the related four immediate surrounding symbols.

9. Last stage. Recovering the indexes to the ISI coefficients:

   Defining the reliable symbols from the set Y, that in magnitude are closer to y_S than y_L, as the set YS. The remaining reliable symbols from the set Y, which are closer in magnitude to y_L than y_S are then defined as the set YL.

   If the set YS has relatively more reliable symbols with immediate surrounding symbols with opposite signs than YL, then the indexes of the smallest in magnitude ISI coefficients, between $|ISI|_{-m,m}$ and $|ISI|_{-n,n}$ become $-1,1$ and the indexes of the largest become -2,2. On the other hand, If the set YL has relatively more reliable symbols with immediate surrounding symbols with opposite signs than YS, then the indexes of the smallest in magnitude ISI coefficient, between $|ISI|_{-m,m}$ and $|ISI|_{-n,n}$ become $-2,2$ and the indexes of the largest become -1,1.

By using the criterion of Eq.(4.32) we can ignore scenario 4.2.2.d In addition it enables to distinguish it from other not reliable symbols. There are some combination of surrounding symbols of not reliable symbols that can fulfil Eq.(4.32). Because their probabilities are small the number of appearance can be checked to find the correct pairs.

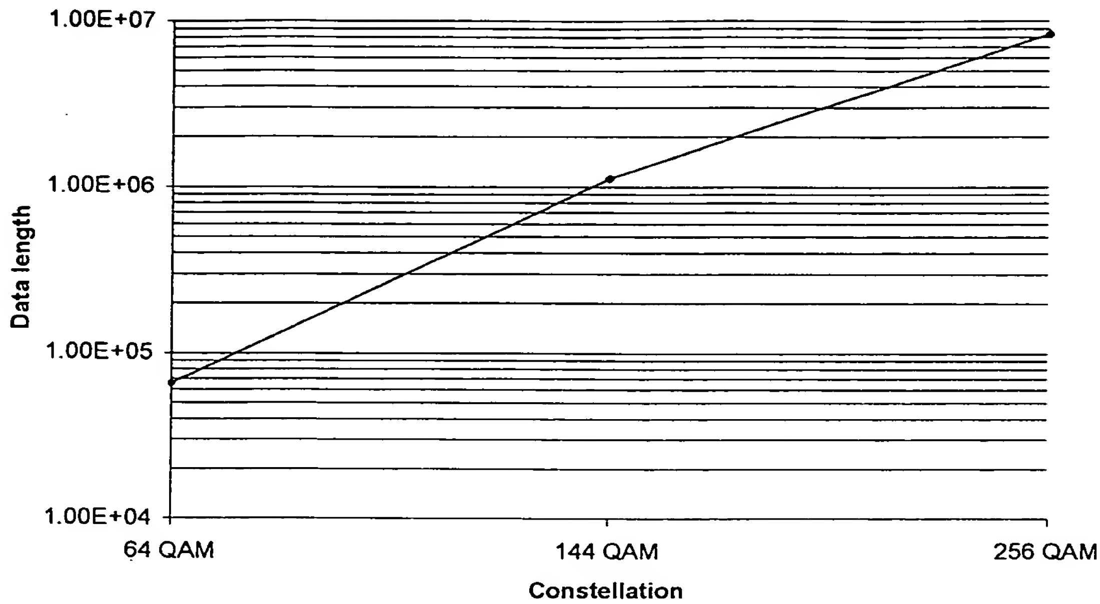

Figure 4.4: Required data length vs. constellations (ISI length – 4). $|ISI| \leq 1$

The length of the required data for the estimation is based on the probability from Eq.(4.11) multiplied by the probability to have reliable symbols just from the first layer in the constellation points. Fig.(4.4) shows the required data length for estimation, with eight reliable symbols (according to the probabilities in 4.2.1.a-c), as a function of the constellation size.

Under this estimation for high SNR there is in effect no error in the ISI coefficients, and in the by-product estimation, the estimation of the constellation points. This accuracy is available because noise is not participating in the process.

Most of the mobile phone standards, like CDMA and UMTS, use as part of their standard, the TDMA (Time Division Multiple Axis) technique. In down link channels where the base-station transmit to the mobile, the data rate is number of times faster (just for the use as random symbols) under TDMA when using the data of the other users. Then the time that is necessary for the estimation process is much shorter. For TDMA with twenty users as an example, where 64,000 symbols are required for the estimation, it takes less then a second to gather the data. Channel estimation under the TDMA standard can be considered then as almost real time process.

4.5 Adaptive properties

Wide range of channels have the property that their parameters are varying with time. Using the "Reliable Symbols" method enables adaptive channel parameters estimations for time varying channels. The limit to the speed the channel can be varying under successful estimation, depends on how fast reliable symbols can be found. For continues channel variation, the different estimations can use a "sliding window". The new members, for the estimation process are then the new reliable symbols with their related surrounding symbols.

The upper limit on the variation speed is a function of the minimum number of reliable symbols needed for estimation ($N_{\min}^{est}$) and the rate of finding a reliable symbol (R_{rel}). Because in the estimation process the processed data has to be under similar channel environment, the upper limit on the variation is determines by the product: $N_{\min}^{est} \cdot R_{rel}$.

The upper limit on the variation is then: The maximum variation where $N_{\min}^{est} \cdot R_{rel}$ transmitted symbols still experience similar channel conditions. Notice that while $N_{\min}^{est}$ depends on the specific parameter estimation, R_{rel} is a function of the probability to find a reliable symbol.

4.6 Simulations

Monte-Carlo simulation was done jointly on the estimation of constellation points and the ISI coefficients. The QAM constellation was used with up to 1024 constellation points. For evaluation of the estimation performance the ISI filter was imposed on the base band symbols. The ISI filters were of the type of symmetric real FIR filter.

Up-to $|ISI| \leq 0.5$ estimations: The theoretical lower limit on the required received data length was confirmed to be enough for estimation for all limits up to 0.5 and ISI length of up to 6 coefficients. For 1024QAM under ISI=(-0.05,0.15,0.15,-0.05) ($|ISI| \leq 0.4$), (Fig.(4.5)) ISI coefficients estimation was done with an error smaller

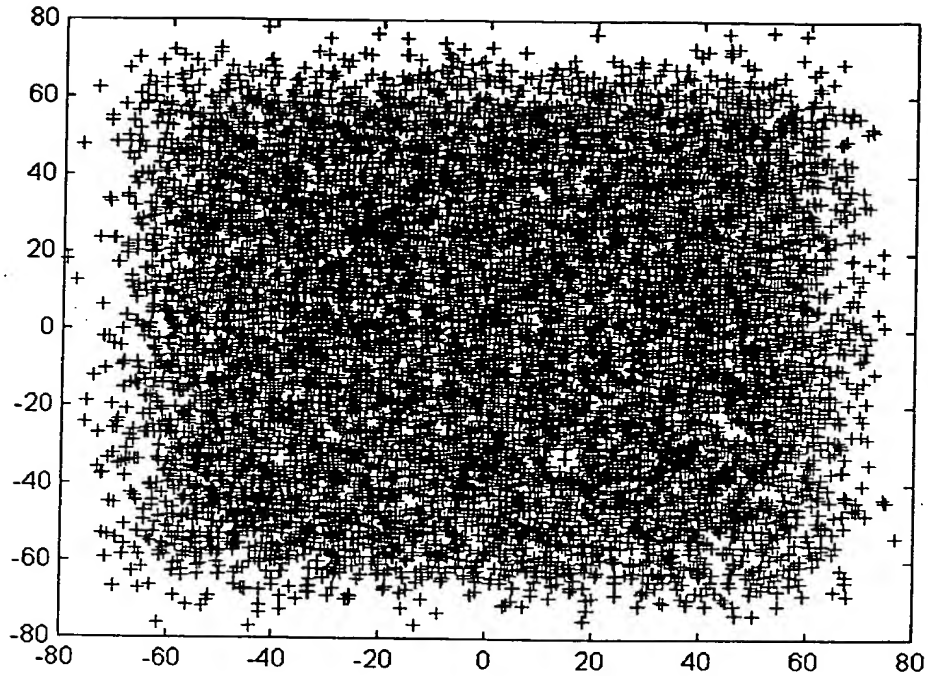

Figure 4.5: 1024QAM under ISI-[-0.05,0.15,0.15,-0.05]

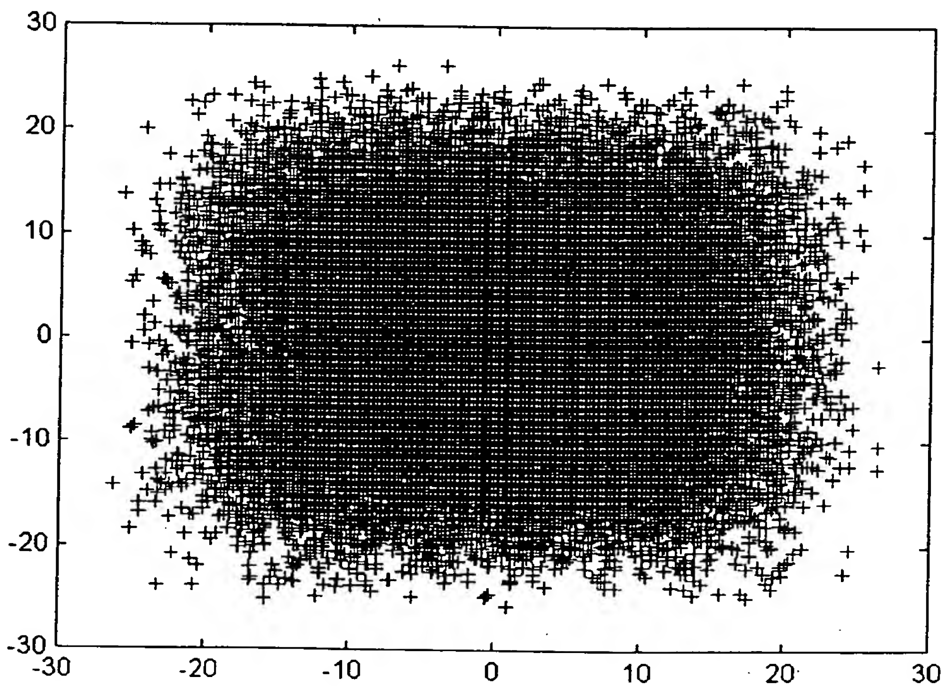

Figure 4.6: 64 QAM under ISI [-0.15,0.35,0.35,-0.15] the edge of "close eye")

than 10% and constellation points estimation with error smaller than 2.5%, for the lowest towards zero constellation point, with 180,000 received data points. For 256QAM with the same ISI and errors, 10,000 points were enough. 64QAM with the same ISI and errors required just 400 received data points. 64QAM with ISI=(0.2,-0.07,0.16,0.16,-0.07,0.02) ($|ISI| \le 0.5$), required 3000 received data points, for the estimation process with about the same errors.

Up to $|ISI| \le 1$ *estimatioins*: The theoretical lower limits of the required data length have been confirmed by simulations. Variety of ISI length-four channels were checked,

with 64,000 received data points. The estimations for both constellation points and ISI coefficients have effectively zero errors. Fig.(4.6) shows 64 QAM with ISI=[-0.15,0.35,0.35,-0.15] ($|ISI| = 1$) I-Q graph, where the errors for both estimations were zero. Although the lower limit is around 64,000 points, for most ISI channels 40,000 received data points was enough for successful estimations.

4.7 Conclusions

We present a new method the "Reliable Symbols" method, which enables successful parameters estimations under high order constellation communication through ISI channels. It is shown that the constellation size is acting as an amplifier on the basic ISI noise level. Reliable symbols can be found under high order constellation, as long as the ISI level does not exceeds the binary open eye level. While there is a limit on the ISI level, there is no theoretical limit on the constellation size. Reliable symbols can be found under real and complex ISI coefficients. Estimations applications, based on "Reliable Symbols" method, for the constellation points and ISI coefficients are derived. The required data length for the channel estimation as a function of the level and length of the ISI, and the constellation size are presented. The estimations were shown to be suitable for a variety of channels, as the telephone and mobile phone channels and for application up to real time tasks. The ISI coefficients estimation reduces to the optimal least square estimation for low order constellation. Simulations results confirm the "Reliable Symbols" method for the channel parameters estimation (blind estimation) under high order constellations. Successful estimation was demonstrated up to 1024QAM.

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CLAIMS

1. A method of high order constellation signal transmission through an ISI channel, by identifying "reliable symbols" which are closer to their received constellation points or scaled transmitted symbols free from ISI noise effect symbols, than to any other received constellation point, and using them to evaluate the channel characteristics.

2. A method according to claim 1 in which comprises identifying a reliable symbol as one which has surrounding symbols each having a relatively lower level, up to some predetermined distance before and after the identified symbol, nominating the identified signal as a "reliable symbol", and repeating the process a number of times along the received signal sequence in order to gather a sufficient number of reliable symbols to enable a proper evaluation of channel characteristics to be achieved.

3. A method according to claim1 or claim 2 further comprising the steps of determining the constellation point locations to establish the channel gain, estimating the ISI coefficients, and setting up an inverse channel filter with corresponding parameters in order to minimise the ISI noise.

4. A method according to claim 3 in which an initial estimation of constellation point locations is made as:

$$\boxed{\hat{P}_i^{\,1} = sign(i)\cdot\frac{\hat{P}^{1}{}_{\max}}{\sqrt{constellation}-1}\cdot(2|i|-1)}$$

Where,

$\hat{P}^{1}{}_{\max}$ - The maximum (in magnitude) reliable symbol.

$i = -k, -k+1, \ldots, -1, 1, \ldots k+1, k.$

where, 2k is the number of constellation points in each axis.

and a final estimation is made as

$$\boxed{\hat{P}_i = \hat{P}_i^1 + (2|i| - 1)\cdot \hat{\tilde{P}}_1}$$

Where,

$$\hat{\tilde{P}}_1 = \frac{1}{2|i|-1}\cdot\left(\frac{1}{|s|}\sum_s \hat{P}_i^1 - y_n^i\right)$$

i = -k, -k + 1,...,-1, 1,...k + 1,k.

where, 2k is the number of constellation points in each axis.

S – The set of detected reliable symbols.

|S| - The number of reliable symbols in the set S.

5. A method according to claim 3 or claim 4 for channels up to the level of $\|ISI\| \leq 0.5$ in which ISI coefficients for the channel are estimated as

$$\boxed{\underline{ISI} = (\underline{\underline{X}}^T \underline{\underline{W}}\,\underline{\underline{X}})^{-1}\underline{\underline{X}}^T \underline{\underline{W}}\,\underline{\Delta}}$$

where,

X – n x k matrix where $n \geq k$, is the ensemble of the surrounding signals for each reliable symnbol.

W – An n x n diagonal weight matrix.

Δ - n x 1 vector, is the metric of each reliable symbol from its origin.

6. A method of high order constallation communication according to claim 3 or claim 4 including a method of estimating ISI coefficients for channels up to the level of $|ISI| < 1$, substantially as herein described.

7. A method of high order constellation communication through ISI channels substantially as herein described with reference to the accompanying drawings.

ABSTRACT

"Increased Data Transmission Bit Rates"

A method of high order constellation communication through ISI channels, by identifying "reliable symbols" which are closer to their received constellation points or scaled transmitted symbols free from ISI noise effect symbols, than to any other constellation point, and using them to evaluate the channel characteristics.

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Block diagram of the proposed method in the communication system:

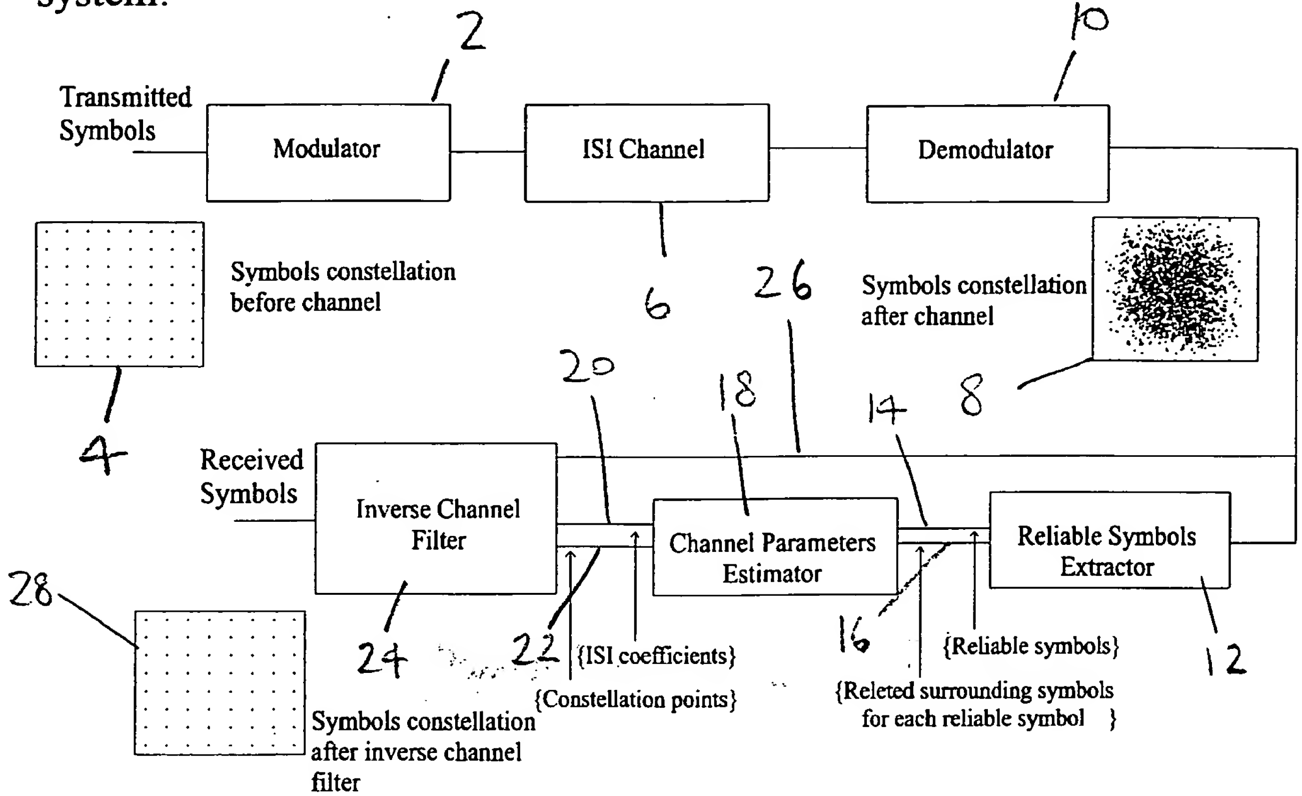


Fig. 1

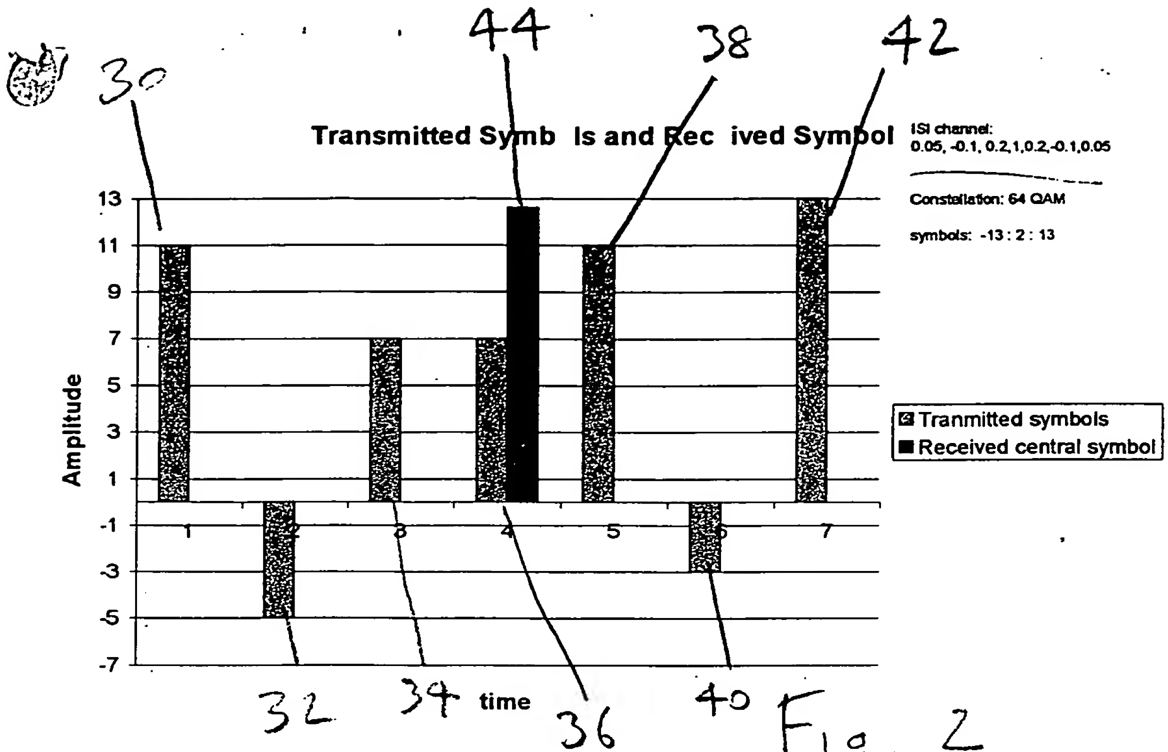
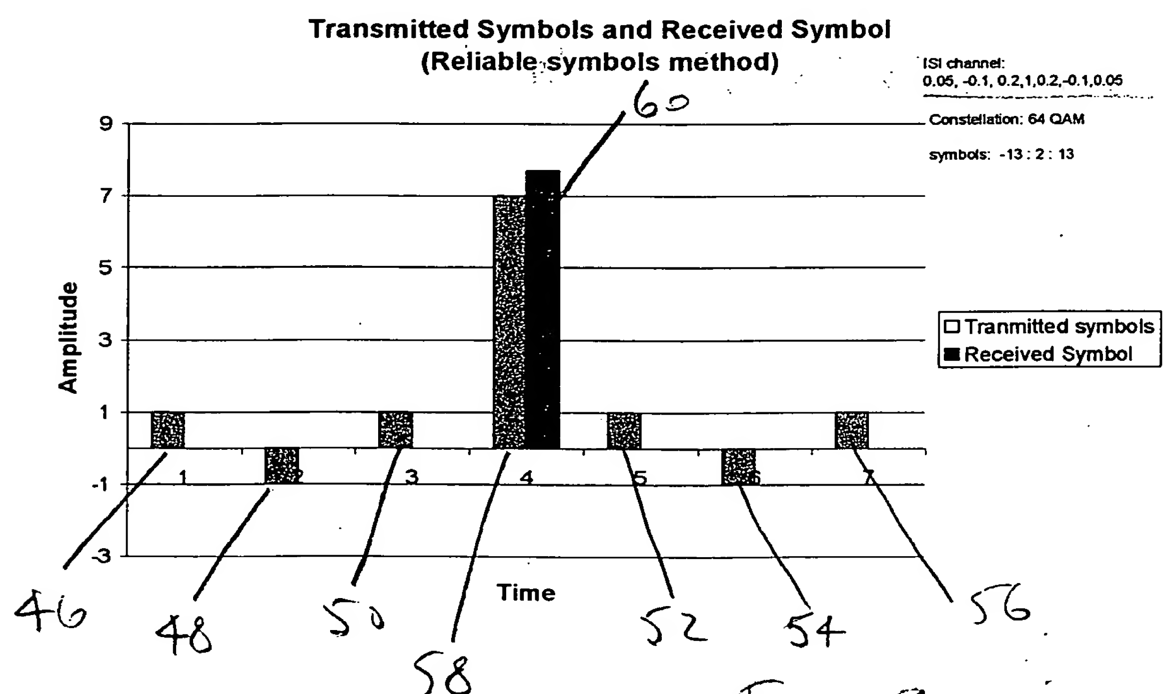

Transmitted Symb ls and Rec ived Symbol
ISI channel: 0.05, -0.1, 0.2,1,0.2,-0.1,0.05
Constellation: 64 QAM
symbols: -13 : 2 : 13
Amplitude
time
Tranmitted symbols
Received central symbol
30
32
34
36
38
40
42
44

Fig. 2


Transmitted Symbols and Received Symbol
(Reliable symbols method)
ISI channel: 0.05, -0.1, 0.2,1,0.2,-0.1,0.05
Constellation: 64 QAM
symbols: -13 : 2 : 13
Amplitude
Time
Tranmitted symbols
Received Symbol
46
48
50
52
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56
58
60

Fig. 3

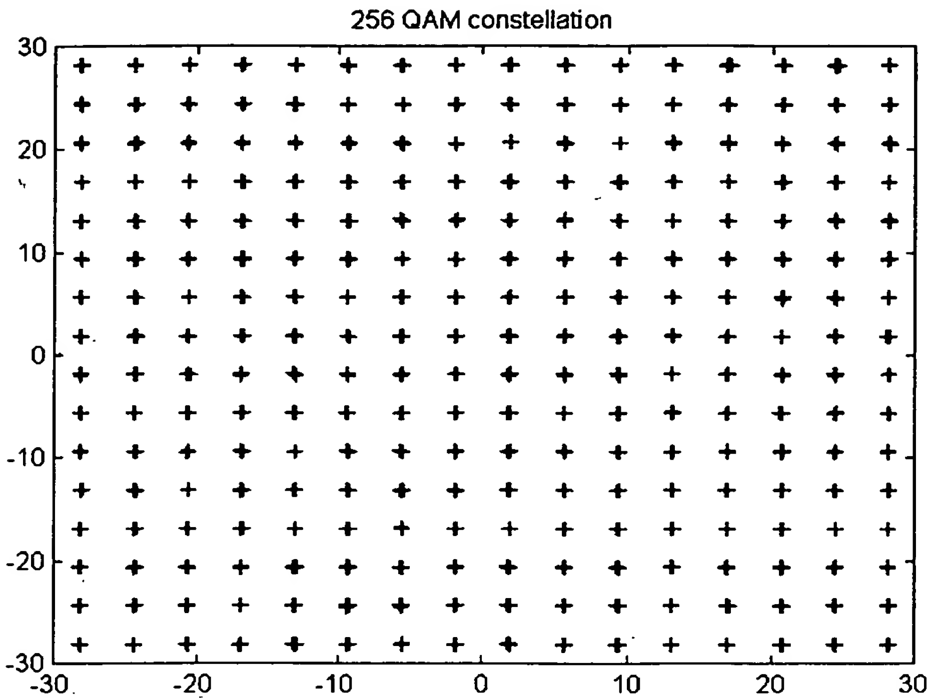

Fig. 4

Constellation Points location Estimation

- *Initial Estimation:*

$$\boxed{\hat{P}_i^1 = sign(i) \cdot \frac{\hat{P}^1{}_{\max}}{\sqrt{constellation} - 1} \cdot (2|i| - 1)}$$

Where,

$\hat{P}^1{}_{\max}$ - The maximum (in magnitude) reliable symbol.

i= -k, -k + 1,...,-1, 1,...k + 1,k.

where, 2k is the number of constellation points in each axis

- *Final Estimation:*

$$\boxed{\hat{P}_i = \hat{P}_i^1 + (2|i| - 1) \cdot \hat{\tilde{P}}_1}$$

Where,

$$\hat{\tilde{P}}_1 = \frac{1}{2|i| - 1} \cdot \left(\frac{1}{|s|} \sum_s \hat{P}_i^1 - y_n^i\right)$$

i= -k, -k + 1,...,-1, 1,...k + 1,k.

where, 2k is the number of constellation points in each axis

S - The set of detected reliable symbols.

|S| - The number of reliable symbols in the set S.

Fig 5

Channel Parameters (ISI coefficients) Estimation

- *LS Estimation:*

$$\boxed{\underline{ISI} = (\underline{\underline{X}}^T \underline{\underline{W}}\,\underline{\underline{X}})^{-1} \underline{\underline{X}}^T \underline{\underline{W}}\,\underline{\underline{\Delta}}}$$

For channels up to the level of $|ISI| \leq 0.5$

where,

X – n x k matrix where $n \geq k$, is the ensemble of the surrounding signals for each reliable symnbol.

W – An n x n diagonal weight matrix.

Δ - n x 1 vector, is the metric of each reliable symbol from its origin.

- *Probability to find Reliable Symbols for different types of channels:*

$$P_R = 2 \cdot \frac{4}{\sqrt{const}} \cdot \left(\frac{2}{\sqrt{const}}\right)^k \cdot (k+1) \qquad : |ISI| \leq 0.5$$

$$P_R = 2 \cdot \frac{6}{\sqrt{const}} \cdot \left(\frac{4}{\sqrt{const}}\right)^k \qquad : |ISI| \leq \frac{1}{3}$$

$$P_R = 2 \cdot \frac{6}{\sqrt{const}} \cdot \left(\frac{4}{\sqrt{const}}\right)^{k-1} \left(\frac{2k+4}{\sqrt{const}}\right) \qquad : |ISI| \leq 0.25$$

Fig. 6